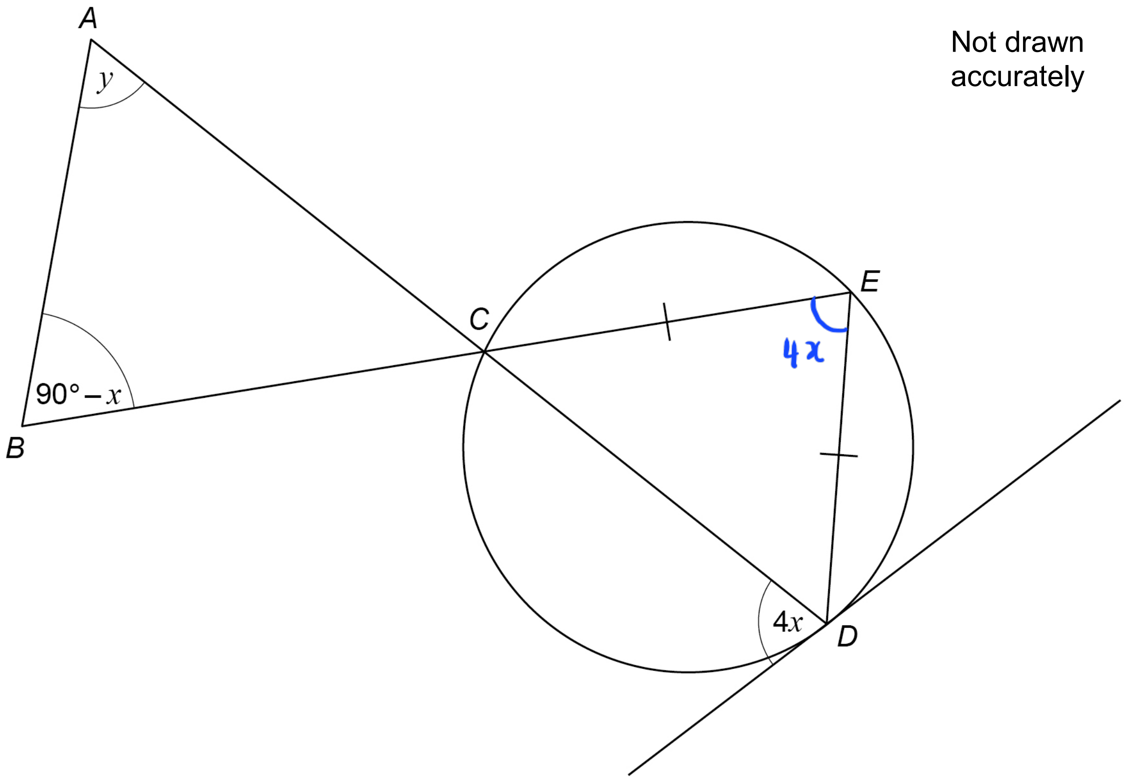
1 C, D and E are points on a circle.

$CE = DE$

The tangent at D is shown.

ACD and BCE are straight lines.

Not drawn accurately



Prove that $y = 3x$

[4 marks]

$CED = 4x$ 1 (alternate segment theorem)

$ACB = 180° - (90° - x) - y$

$= 90° + x - y = DCE$ 1 (vertically opposite angles are equal)

1

$DCE = \dfrac{180 - 4x}{2}$ (base angles of isosceles are equal)

$\dfrac{180 - 4x}{2} = 90 + x - y$ 1

$180 - 4x = 180 + 2x - 2y$

$6x = 2y$

$y = 3x$ (shown)

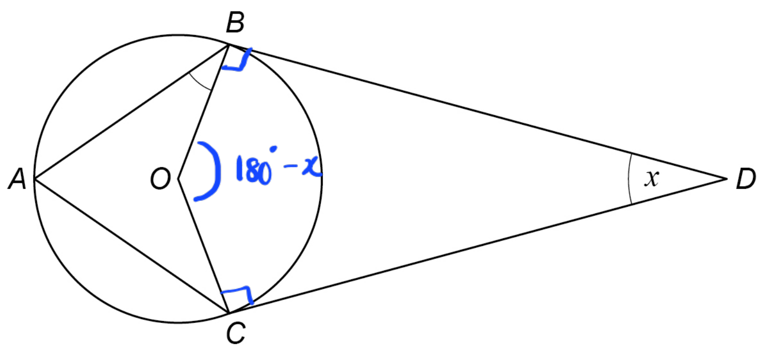
2 A, B and C are three points on the circumference of a circle, centre O.

BD and CD are tangents to the circle.

$ABDC$ is a kite.

Angle BDC is x

Not drawn accurately



Prove that angle ABO is $45° - \frac{x}{4}$

[4 marks]

$OBD = OCD = 90°$ (1)

(tangent meets the radius at 90°)

BOC (obtuse) $= 180° - x$

(angles in a quadrilateral add up to 360°)

$BAC = \frac{180° - x}{2}$ (1)

(angles at circumferenco is half angles at centre)

BOC (reflex) $= 360° - (180° - x)$ (1)

$= 180° + x$

(angles around a point add up to 360°)

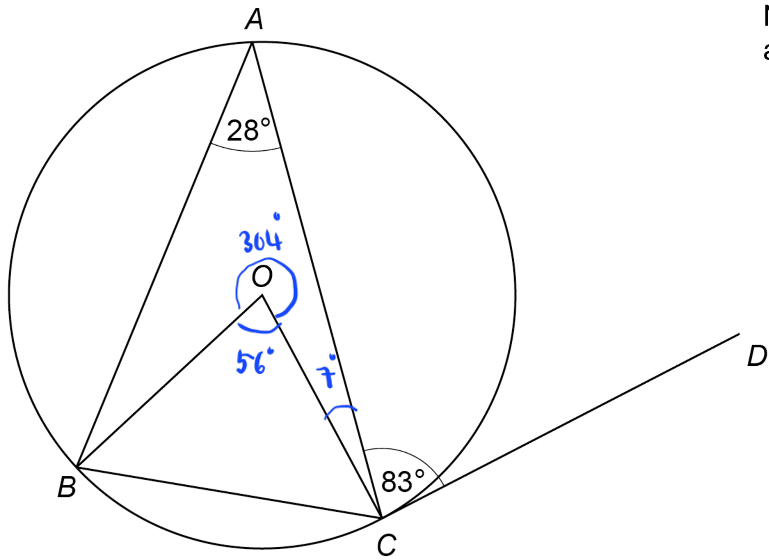
$ABO + ACO = 360° - (180 + x + 90° - \frac{x}{2})$

$= 90° - \frac{x}{2}$ (1)

$ABO = \frac{1}{2}(90° - \frac{x}{2}) = 45° - \frac{x}{4}$ (proved)

3 A, B and C are points on a circle, centre O.

DC is a tangent to the circle.



Not drawn accurately

Show that angle ABO : angle $ACO = 3 : 1$

[5 marks]

ACO = 90 − 83 = 7° 1

BOC (small) = 2 × 28° = 56° 1

BOC (large) = 360° − 56° = 304° 1

ABO = 360° − 304° − 7 − 28°

= 21° 1

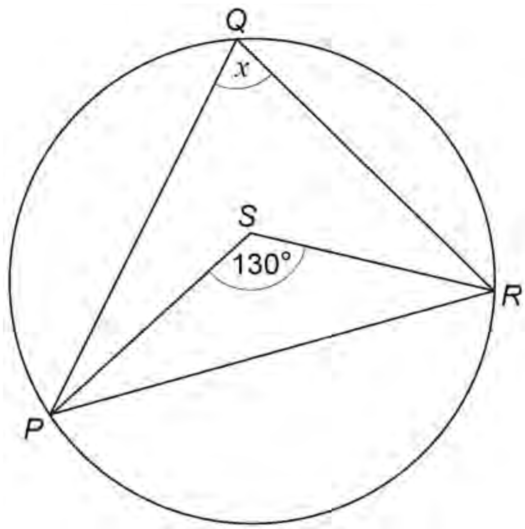
ABO : ACO = 21 : 7 ÷7

3 : 1 1

4 (a) P, Q and R are points on a circle.

S is a point inside triangle PQR.

Not drawn accurately



Assume that S is the centre of the circle.

Work out the size of angle x.

[1 mark]

$x = \frac{130}{2} = 65$ 1

$x =$ 65 °

4 (b) In fact, the centre of the circle is on PS but **not** at S.

What does this mean about the size of angle x ?

Tick **one** box.

[1 mark]

☐ It is the same as the answer to part (a)

☑ 1 It is greater than the answer to part (a)

☐ It is smaller than the answer to part (a)

☐ It is impossible to tell

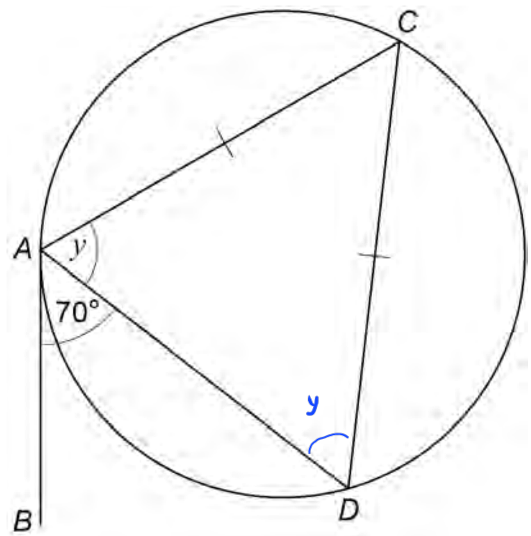
4 (c) For a different circle,

AB is a tangent at *A*

C and *D* are on the circumference of the circle

AC = *CD*

Not drawn accurately



Here is Simon's method to work out the size of angle y.

Angle *ADC* = 70° (alternate segment theorem)
Therefore y = 70° (angles in an isosceles triangle)

Is he correct?

Give a reason for your answer.

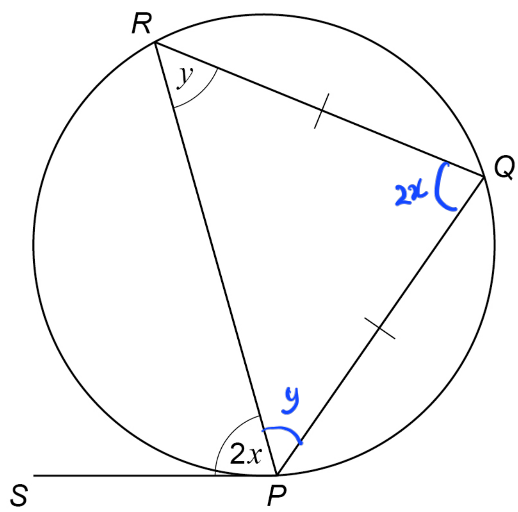
[1 mark]

No. He is wrong. angle ADC is not 70°. 1

5 P, Q and R are points on a circle.

SP is a tangent to the circle.

$RQ = PQ$



Not drawn accurately

Prove that $y = 90° - x$

[4 marks]

PQR = SPR = 2x (1)

(alternate segment theorem

RPQ = PRQ = y (1)

(base of isosceles triangle are equal)

2x + 2y = 180° (1)

(angles in a triangle add up to 180°)

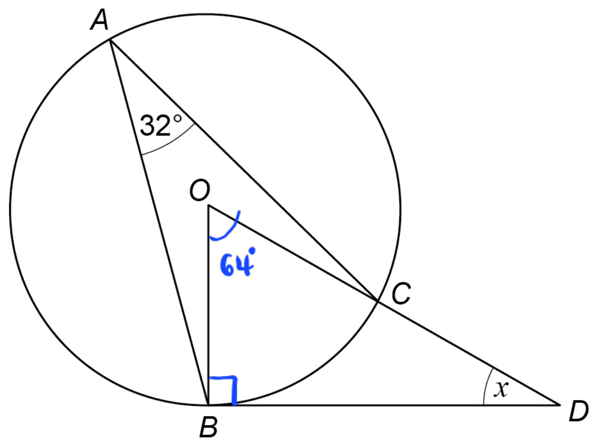
2y = 180° - 2x

y = 90° - x (1)

6 A, B and C are points on a circle, centre O.

BD is a tangent to the circle.

OCD is a straight line.



Not drawn accurately

Work out the size of angle x.

[3 marks]

BOD = 32 × 2 = 64° 1

x + 64° + 90° = 180°
1

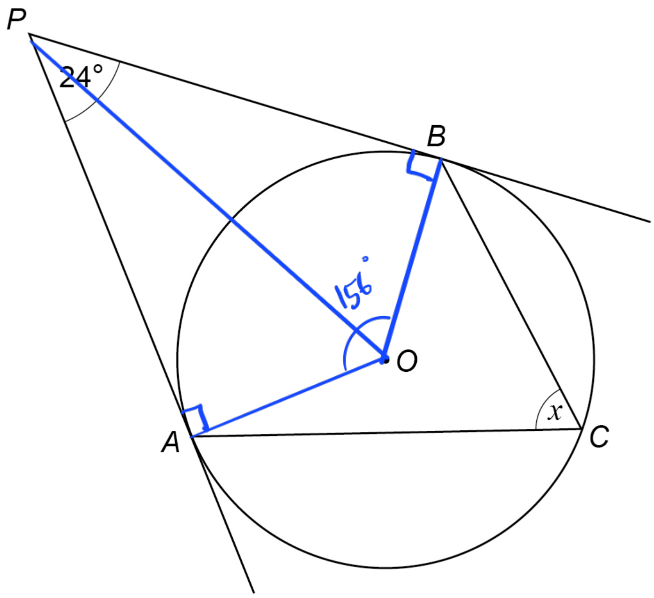
x = 180° − 90° − 64°
= 26° 1

$x =$ 26 degrees

7 A, B and C are points on a circle, centre O.

AP and BP are tangents to the circle.

Not drawn accurately



Work out the size of angle x.

[3 marks]

angle PBO = angle PAO = 90° ①

angle AOB = 360 − 90 − 90 − 24 = 156° ①

$x° = \frac{156°}{2} = 78°$ ①

(angle at the centre is twice angle at circumference)

Answer 78° °